# Provincial Growth and Development Through The Equitable Sharing of Infrastructure Finance[1]

by

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**Abstract**

Since 1999 the Financial and Fiscal Commission has worked on the development of a capital grant model for the allocation of intergovernmental infrastructure funding to Provinces. The aim of the model is to allow policy-makers at the national sphere of government to distribute a pool of funds on the basis of relative need to the Provinces for capital expenditures for services such as education, health, welfare, transport and housing. In this paper we describe some of the background to this work, outline its rationale, describe the formulas underlying the grant model in detail and, finally, we provide preliminary simulations.

***Keywords:*** public capital, economic growth, equity, historical backlogs, cost disabilities.

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## 1. Introduction

Between 1999 and 2004 the Financial and Fiscal Commission, in conjunction with technical advisors from Australia and assistance from *AusAid*, developed a scheme to allocate capital grants to the Provinces on the basis of relative need for public capital. The aim of the scheme was to allow the National Government to supplement Provincial capital expenditures directed at infrastructure for important services such as education, health, transport housing and welfare. The scheme, and the research underlying it, has two key components: a conceptual part that sets up the formulas used to estimate needs and allocate grants, and a computer simulation model which allows the policy maker to estimate Province-specific grants under various assumptions about certain policy parameters, such as the amount of money to be allocated in each period.

The purpose of this paper is to explain the rationale behind the development of the grant scheme, provide a detailed discussion of the conceptual model and the input database constructed for the project and, finally, to present preliminary simulation results. The structure of the paper is as follows. Section 2 presents the rationale for a capital grant scheme while Section 3 provides an analysis of capital cost disabilities. The grant model itself is analysed in Section 4. In Section 5 we discuss the input database and the computer simulation model, and present some example grant simulations that should, at this stage, only be considered as preliminary. Brief conclusions are presented in Section 6.

## 2. Rationale for Capital Grants

The case for capital grants to the Provinces in South Africa rests on three arguments, the State's constitutional obligation to provide basic services, economic growth and equity of access to these basic services. Each is discussed briefly below.

### *2.1 The Provision of Constitutionally Mandated Basic Services (CMBS)*

The Bill of Rights (Chapter 2) of the South African Constitution obliges all three spheres of government to progressively provide certain basic socio-economic services subject to norms and standards and the availability of resources. The provision of many of these services requires the construction, maintenance and operation of appropriate and adequate infrastructure.

The Bill of Rights covers a wide range of services and rights that require physical infrastructure. In summary they include the right to a healthy, safe and sustainable environment; land reform and restitution of property; access to adequate, housing, health care, food, water and social security; every child's right to basic nutrition, shelter, health care and social services; the right to basic and further education and training.

The right to education poses a considerable challenge to government. The constitution says in Section 29(2), "Everyone has the right to receive education…in public educational institutions where the education is reasonably practicable. In order to ensure the effective access to, and implementation of, this right, the state must consider all reasonable alternatives,…, taking into account-

(a) equity

(b) practicability; and

(c) the need to redress the results of past racially discriminatory laws and practices."

Other rights and services that require infrastructure provision are the right to freedom and security; the right to access to information; the right to access to courts and, the right to just administrative action.

The Bill of Rights does, however, indicate that these rights may be progressively achieved by the State through reasonable measures within available resources.

In Chapter 13 Section 214 (2) (d) of the Constitution National Government is enjoined to "… to ensure that the provinces and municipalities are able to provide basic services and perform the functions allocated to them". In addition, section 214 (2) also lists several other principles that National Government must consider when allocating revenue to provinces and municipalities. These encompass fiscal capacity and efficiency;

developmental and other needs; intra and inter-regional economic disparities; nationally legislated obligations and, the need for stable and predictable allocation of

Given the considerations noted above, and the problem of scarce fiscal resources that have to be shared equitably among and within three spheres of government, a capital grant model and formula does give national government an objective mechanism for progressively financing the physical infrastructure necessary for the delivery of constitutionally mandated basic services.

### *2.2. Efficiency and Economic Growth*

The past ten years of democracy and intergovernmental fiscal relations in South Africa has resulted in improvements in the provision of basic services to millions of citizens hitherto marginalized. However, despite these improvements the devastating legacy of apartheid has resulted in a massive backlog of physical and social infrastructure required to support the provision of basic services. Existing infrastructure is woefully inadequate and widely dissimilar across provinces.

In an attempt to address these backlogs National Government incorporated a notional 'backlogs' element of 3% within the equitable sharing funding formula for provinces. To date there is no evidence to suggest that such a mechanism has had the desired effect. There are several possible reasons for this failure. Capital spending by provinces was inadequate to simultaneously address growing infrastructure backlog needs and other recurrent capital expenditures. National Government's annual IGFR Reviews indicate that between 1996/97 and 1998/99 there was an overall decline in the provincial funding of capital in the order of 5.3% to 4.2%. This problem is exacerbated by expenditure pressures to finance core non-capital basic service costs that compete for a share of the same pool of the funds. As the nine provinces have inherited widely differing levels of public infrastructure, the impact on the delivery of services is uneven. Non-existent own revenues and limits to accessing to capital markets impose further constraints on the provinces' ability to fund capital expenditures.

In 2001 National Government accepted an FFC recommendation to provide provinces with a conditional grant to address infrastructure backlogs. Indications over the past three years suggest that provinces experience many difficulties in accessing this pool of funds earmarked for financing infrastructure in general and infrastructure backlogs in particular. In this context it became necessary for the FFC to develop a capital grants scheme for the objective and equitable allocation of funds to address provincial infrastructure needs specifically.

The authors of this paper believe that the overall amount of 'social capital' available for the provision of important services such as education and health is insufficient in South Africa. This appears to be so if one looks at international benchmark comparisons of the amount of capital available for the provision of these services in South Africa. For example, Petchey and Levtchenkova (2002) estimate that if one uses the amount of capital available to supply education services in Australia as a benchmark, then South Africa has a backlog in education capital alone of some R108 billion. A comparable backlog estimate for health is in the order of R60 billion[6]. These backlogs arise because South Africa is an economy in transition. Indeed, one might argue that applying the standards of a high-income economy in South Africa is not entirely appropriate. Nevertheless, such an exercise does highlight the magnitude of the extent of the deficiency in public sector capital at an aggregate level when one uses an international benchmark.

One might, therefore, argue that there is a case for a general increase in public capital formation to raise the overall amount of capital dedicated to supplying the basic public services of education, health, welfare, transport and housing. It is well known that there is a link between public capital formation, the creation of human capital and economic growth. This is a lesson that has been put to good use in Asian economies, such as Singapore and Malaysia, where public capital formation (particularly in education) has

[6] Petchey, J.D and S. Levtchenkova (2002). A Policy Model for Reducing Public Capital Backlogs in Transitional Economies with Regions: The Case of South Africa. School of Economics and Finance, Curtin University of Technology, Perth, WA, Australia.

been an integral part of economic growth strategies for many years. Thus, there is a 'growth' rationale for increased public capital formation.

### *2.3. Domestic Backlogs and Equity of Access to Services*

The distribution of the existing public capital stock across Provinces is also highly uneven. For example, as a part of this research project the authors have estimated the per capita value of the public capital stock in each Province for education, health, welfare, transport and housing (Table 1). It is clear that there is a very wide variation in this indicator across Provinces, with, for example, Northern Cape having nearly 2.5 times the per capita public capital of Limpopo. Much of this geographic inequity in the availability of public capital is due to past policies of apartheid that favoured certain regions. The implication is that some Provinces have 'backlogs' for the provision of basic public services and this creates widely variable access to services across Provinces.

This suggests that there is an equity case for some 'equalisation' of the per capita capital stock between Provinces, perhaps by eliminating the historically created backlogs, to ensure that there is better equality of access for all citizens to services. Such equalisation would also contribute to the goal of social stability.

**Table 1: Estimated Capital Stock Per Capita by Province (for the aggregate of Education, Health, Transport, Welfare and Housing), 2002**

| | Real Capita Stock (1995 prices) R Million | Population | Real Capital Stock per Capita (R thousands) |
|---|---|---|---|
| Western Cape | 6,912 | 4,310,000 | 1,604 |
| Eastern Cape | 4,058 | 7,130,000 | 570 |
| Northern Cape | 1,743 | 890,000 | 1,958 |
| Free State | 3,086 | 2,860,000 | 1,079 |
| Kwa Zulu Natal | 17,608 | 9,210,000 | 1,949 |
| North West | 6,018 | 3,660,000 | 1,644 |
| Gauteng | 12,832 | 8,110,000 | 1,582 |
| Mpumalanga | 5,405 | 3,160,000 | 1,710 |
| Limpopo | 4,745 | 5,840,000 | 812 |
| South Africa | 62,407 | 45,170,000 | 1,382 |

Sources: (i) Capital stock by Province: Data supplied as a special request from the SA Reserve Bank.
(ii) Population by Province: Statistics South Africa.

However, we must face the reality that the important social services are provided by the Provinces, and the fact that these Provinces do not have the tax base or access to capital markets to finance an expansion in public capital formation. Moreover, without these policy instruments, Provinces would find it extremely difficult to leverage financial capital from the private sector for such spending.

This means that, at least in the medium term, any expansion of spending on public capital would need to occur via a National Government grant. Indeed, the aim of this project, and previous work undertaken by the Financial and Fiscal Commission on their existing grant model, has been to develop such a mechanism, namely, a model that allocates grants to the Provinces on the basis of need.

## 3. Provincial Cost Disabilities

The first step in developing the scheme for allocating capital grants is to examine the 'cost disabilities' of the Provinces. Cost disabilities attempt to capture factors that cause variations in the cost of providing public services between Provinces and which imply that a Rand spent on capital will have different impacts depending on where it is spent. The discussion here has two parts. In the first, we present a brief overview of international experience with cost disabilities, and in the second, we develop our approach to constructing and measuring disabilities.

### *3.1 International Experience*

Many developed countries[7] have grants from national governments to provinces or states that incorporate the notion of cost disabilities. A good example here is Switzerland where differences in the cost of providing services in mountainous areas are taken into account in equalisation grants. For example, one disability proxy measures the relative significance of agricultural land above 800 metres. The logic here is that it is relatively more expensive to provide government services to farms in more mountainous regions. Thus, in order to provide 'one unit' of service in a mountainous region, one has to spend more than what would be required in a flat region with easier access. The Swiss also have proxies for population density in their allocation model. Again, the logic is that it is relatively more expensive to provide services to a geographically dispersed population. In Japan too, cost disabilities are included in the scheme that allocates equalisation grants to local governments. The types of disabilities accounted for include population density, population growth, climate, area and geography, degree of urbanisation, and industrial diversification. The UK also takes account of cost disabilities when allocating grants from the national government to local jurisdictions. Interestingly, these disabilities are constructed using a cross-section regression analysis.

However, when it comes to allowing for and estimating cost disabilities for grant programs, Australia is generally seen as the benchmark country. In Australia, the

[7] Examples include the US, UK, Japan, Switzerland, Australia and Germany. See Ter-Minassian, T (1997). *Fiscal Federalism in Theory and Practice*, International Monetary Fund, Washington DC.

Commonwealth Grants Commission has an extensive methodology for calculating cost disabilities for inclusion in its fiscal equalisation model which is used to allocate in excess of $A50 billion to the states annually. The key disabilities assessed are age/sex composition of the state's population, population dispersion, various indexes which measure the scale of provision of services and socio-economic indicators such as the number of indigenous people as a proportion of state population.

### *3.2 Our Approach*

We define the capital cost disability for Province i in period t as

$$\gamma_{i,t} = e^{\phi_{i,t}} \qquad \textit{(Capital Cost Disability)} \qquad (1)$$

where

$$\phi_{i,t} = \sum_{i,t,j=1}^{J} \beta_{i,t,j} D_{i,t,j} \qquad \textit{(Disability function)} \qquad (2)$$

is a capital cost *disability function* for Province i in period t and[8]:

(1) $D_{i,t,j}$ is the percentage deviation, for Province i, of the $j^{th}$ disability measure from the mean value of the disability measure for all Provinces (for period t).

(2) $0 \le \beta_{i,t,j}$ is a parameter that captures the impact of the percentage deviation of the $j^{th}$ disability from its mean value on the value of the disability function (in period t).

(3) $D_{i,t,j} = \left(X_{i,t,j} - \overline{X}_{t,j}\right) / \overline{X}_{t,j}$, where $X_{i,t,j}$ is the value of the $j^{th}$ disability measure for Province i and $\overline{X}_{t,j}$ is the mean value of the $j^{th}$ disability measure for all Provinces i = 1,....,9 in period t.

The construction of equation (1) implies that the capital cost disability is a non-linear (exponential) function of $\phi_{i,t}$. An advantage of this is that the disability will be

[8] For generality here we allow beta and the mean deviations to be Province, period and disability specific. Due to data limitations, in the simulations we allow beta and the mean deviations to be disability specific but the same across Provinces and time periods.

normalised around one; a useful feature when incorporating the cost disability into the capital grant model. One could use alternative functional forms, for example, one that expresses the cost disability as a linear function of $\phi_{i,t}$, yet is still normalised around one. However, we have no a-priori information on whether the capital cost disability is a linear or non-linear function of $\phi_{i,t}$ so using the exponential form seems reasonable for now.

The parameter $\beta_{i,t,j}$ in the capital cost disability function is assumed to be a policy parameter. One can see from equation (2) that the choice of $\beta_{i,t,j}$ will have a considerable impact on the relative importance of the different disabilities incorporated into the disability function, and on the overall magnitude of the value of the disability function itself. Therefore, we need to be careful in our choice of $\beta_{i,t,j}$. Ideally, we would like this to be determined from an econometric study that yields estimates of how important each disability is in determining the value of $\phi_{i,t}$. However, in the absence of such information for South Africa we assume these values to be policy parameters, which are disability but not Province specific[9].

The capital cost disability function can have three different signs. First, if $\phi_{i,t} = 0$ then the weighted sum of the percentage deviations for Province i is exactly equal to zero. In this case, $\gamma_{i,t} = e^0 = 1$ and as we shall see below, the capital cost disability has no influence on the Province's grant. Second, if $\phi_{i,t} \leq 0$ then the weighted sum of the percentage deviations is negative implying that $0 \leq \gamma_{i,t} < 1$ (a relatively low cost Province). As will be seen below, if $0 \leq \gamma_{i,t} < 1$ then the disability will tend to reduce a Province's grant below what it would otherwise be. Finally, if $\phi_{i,t} \geq 0$ then the weighted sum of the percentage deviations is positive, implying that the Province has relatively

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[9] An econometric study of this nature has been undertaken for Australia by Petchey, J.D., Shapiro, P., MacDonald, G, and P. Koshy (2000). Capital Equalisation and the Australian States. *The Economic Record.* 76:232. March. 32-44.

high costs, and $\gamma_{i,t} > 1$. Here, the effect of including disabilities is to raise a Province's grant above what it would otherwise be.

At least three cost disabilities might be constructed for South Africa. The first is population dispersion. Consider, for example, a geographically large Province with a dispersed population. The cost of providing a school or hospital in the remote regions of such a Province is higher than the cost of providing the same school or hospital in an urban region of a Province with a predominately city-based population. This is because, to provide a school or hospital in a remote location, it is also necessary to incur the cost of providing access roads, extending electricity and water systems and other infrastructure. Because of such a 'population dispersion disability' the per unit cost of the flow of capital services in such a Province may be relatively high.

A second example of a capital cost disability that may be relevant in South Africa relates to a health disability indicator. If a Province has a relatively high incidence of a health disability factor (such as TB; malnutrition; HIV/AIDS; etc.) in its population, then the cost of each unit of health service may be high relative to a Province with a relatively lower incidence of these debilitating diseases. This is because such diseases require considerable resources to manage.

Third, the cost of providing a given unit of public service output may be higher in Provinces with relatively poor socio-economic indicators. For example, the cost of achieving given educational and health outcomes for people from poor families may be higher than the cost of achieving the same educational or health outcomes for people from richer backgrounds. Provinces with more poor people might, therefore, be expected to incur higher costs in achieving given health and educational outcomes. Thus, we suggest that some general measure of socio-economic disability be included in the disability function.

In summary, it is suggested that at least the following disabilities be developed:

1. Population dispersion.
2. Debilitating disease factors (e.g., TB; malnutrition, HIV/Aids).
3. Socio-economic factors.

**4. The Grant Scheme**

We now turn to the main part of the paper: the development of the formulae for the capital grant model. The following discussion develops the capital grant scheme formulas that are used in allocating grants to the Provinces. These formulas will incorporate the cost disabilities discussed above. Before commencing, it is useful to note that we use the following variable and parameter definitions:

$K_t =$ Value of the actual capital stock (1995 constant prices) for all Provinces in South Africa in period t for the constitutionally mandated basic services (education, welfare, health, transport and housing).

$K_{i,t} =$ Value of the actual capital stock (1995 constant prices) for Province i in period t for education, welfare, health, transport and housing.

$P_{i,t} =$ Population of Province i in period t.

$P_t =$ Population of South Africa in period t.

$CP_t =$ Capital grant pool in period t (derived from macroeconomic forecasts with policy input or could be given as a policy parameter).

$0 \leq \delta_t \leq 1 =$ Proportion of the capital grant pool allocated to domestic backlogs (policy parameter).

$0 \leq \gamma_{i,t}$ is the capital cost disability for Province i in period t.

$\phi_{i,t} =$ the disability function for Province i in period t.

It is convenient to develop the model by first examining period one, and then going on to discuss periods two and three. It should be noted that each period could coincide with one year, or a number of years. For example, we could consider period one to coincide with the three years of the MTEF cycle.

### *4.1 Period One*

Maybe We noted above that the most important reasons for a capital grant were the need to raise the overall level of public capital formation for the delivery of important public services, and the issue of access to these services. For this reason, our grant scheme is divided into two components: a per capita component to address longer run economic growth and a domestic backlogs component designed to address inequality of access to services created by the historical backlogs. The formulas for each component (for period one) are developed below, starting with the domestic backlog part.

#### *(a) The Domestic Backlog Grant*

In period one, some amount of the grant pool, $CP_1$, is 'top-sliced' for allocation to the Provinces based on domestic backlogs. The amount allocated to this component of the scheme is $CP_1 \cdot \delta_1$ where $0 \leq \delta_1 \leq 1$ is a policy parameter.

To determine the allocation of this top sliced pool in period one, we need a measure of the amount of capital that we would like each Province to have for the provision of constitutionally mandated basic services. There are several ways to do this. One would be to use per capita minimum standards developed for each of the services to estimate the amount of capital that a Province would require to meet such standards. Comparison of these standards against a Province's actual capital stock would yield a measure of the domestic backlog for each Province. Though appealing, the problem with this approach is that, as far as we are aware, there are no commonly agreed upon and consistent minimum standards for all the Provincial services of interest. Thus, it is not feasible to use this approach at present though it may be at some stage in the future if adequate measures of minimum standards become available.

We propose using a simple per capita average as the standard against which to compare each Province in deciding on whether it has a 'domestic backlog', that is, a shortage of capital relative to other Provinces. The first step in constructing this standard is to note that the (per capita) stock of Provincial capital is

$$\frac{K_1}{P_1} \qquad \textit{(Standard capital stock)} \qquad (3)$$

In the following analysis we apply this in each Province to derive the amount of capital that a particular Province needs in order to meet the per capita average. For this reason, we refer to it from now on as the 'standard capital stock'.

The use of a per capita average in this way must be treated with some caution since it may 'mask' service specific backlogs within a region. For example, consider a Province with large deficiencies in infrastructure for housing but more than adequate infrastructure for all of its other services (it was pointed out in discussions that Gauteng was such a Province). Our aggregate measure of the Province's capital stock, together with the use of an average for the standard, may indicate an overall 'surplus' of capital for the Province, thus masking the fact that there is a deficiency for one particular service[10]. One can see from this example, that the use of an average to calculate a standard is, therefore, a relatively simple way to measure needs.

However, as we will see below, we are proposing that all Provinces also receive a per capita grant from some portion of the grant pool. To an extent, this allocation will overcome the masking effect resulting from the use of a simple average for calculating a domestic standard. A Province such as Gauteng, which as we will see below, does not have a domestic backlog when estimated using an average, will still receive an equal per capita allocation for capital spending from the grant pool. A policy decision would need to be made within the Province to use this equal per capita allocation to address housing backlogs.

[10] Unfortunately, it is not really feasible to convert 'surplus' infrastructure used for providing, say, health, into housing infrastructure.

It should also be remembered that there are substantial benefits from using an average to set the domestic standard. It has intuitive appeal to notions of equity or fairness and is feasible, since as noted above, there is no other way, at present, of constructing a domestic-based standard for capital. Using an average as the standard also 'de-politicises' the process of estimating the standard since it is based on a mathematical calculation with no potential for 'rent seeking' behaviour.

Next, we define the aggregate amount of capital that each Province requires in order to achieve the standard. For Province i in period one this is just the per capita standard from (3) multiplied by the Province's population. However, we also need to take into account the fact that Province i has a cost disability, which may be less than or more than one. If more than one, the implication is that Province i would need more than the standard amount of capital in order to provide the standard amount of services from that capital. If less than one, the implication is that the Province would need less than the standard in order to provide the standard amount of services.

Our measure of the amount of capital needed in Province i to achieve the standard, taking account of cost disabilities, is:

$$\left(\frac{K_1}{P_1} \cdot P_{i,1}\right) \cdot \gamma_{i,1} \qquad \textit{(Desired Capital Stock)} \qquad (4)$$

Thus, (4) is the amount of capital needed in Province i if that Province is to achieve the South African standard taking into account its capital cost disability in period one. Whether a Province has a domestic backlog depends on how its actual capital stock deviates from the disability-adjusted standard capital stock for that Province. This can be calculated for Province i, and period one, as follows:

$$B_{i,1} = \left(\frac{K_1}{P_1} \cdot P_{i,1} \cdot \gamma_{i,1} - K_{i,1}\right) \qquad \textit{(Historical Backlog)} \qquad (5)$$

Provinces with $B_{i,1} > 0$ (a positive capital backlog) have a deficiency in their capital stock in the sense that it is inadequate to achieve the domestic standard, as determined from (4). Provinces assessed with $B_{i,1} < 0$ have a negative backlog based on the domestic standard. Their capital stock exceeds what is required to achieve the standard. Finally, $B_{i,1} = 0$ implies that a Province has no domestic backlog. The capital stock in such a Province is equal to what it needs to achieve the South African average, adjusted for cost disabilities.

The top-sliced pool is allocated among those Provinces with a positive domestic backlog according to the following formula that automatically ensures that the sum of the domestic backlog grants is equal to the top-sliced pool:

$$G_{i,1}^{D} = \left\{ \begin{array}{ll} \dfrac{B_{i,1}}{\sum_{i:B_{i,1}>0} B_{i,1}} (\delta_1 \cdot CP_1) & \forall \ \ B_{i,1} > 0 \\ 0 & \forall \ \ B_{i,1} \leq 0 \end{array} \right\}, \quad \textit{(Historical backlog grant)} \qquad (6)$$

where $G_{i,1}^{D}$ is the domestic backlog grant to Province i in period one. Only those Provinces with a positive domestic backlog receive this grant. Provinces with $B_{i,1} \leq 0$ receive nothing from this part of the grant pool in period one.

***(b) The Per Capita Grant***

Recall that our other rationale for a grant is to raise future economic growth. Therefore, the remaining portion of the pool, $(1-\delta_1)$, is allocated on a per capita basis to all Provinces to help them overcome their overall shortage of capital relative to international standards. The aggregate amount remaining in the pool for this grant is:

$$CP_1(1-\delta_1). \qquad \textit{(Pool available for per capita grant)} \qquad (7)$$

As noted, the value of the parameter $\delta_1$ is chosen by the policy maker. The greater is the value of $\delta_1$, the smaller is the allocation to the efficiency-based grants, and the greater is the allocation to the historical domestic backlogs. Conversely, the lower is the value of $\delta_1$, the smaller is the amount allocated to equality of access component (the domestic backlogs) and the greater is the allocation of funds to addressing efficiency. From this, one can see that the policy maker has the discretionary power to decide how quickly the domestic backlog is eliminated. Also, once the historical domestic backlogs are eliminated and there is more equality of access to services, the parameter will be set equal to zero, implying that all of the pool is allocated to the growth component.

A Province's aggregate grant from this component of the pool in period one is:

$$\left(\frac{CP_1(1-\delta_1)}{P_1}\cdot\frac{P_{i,t}}{P_t}\right)\cdot\gamma_{i,1} \tag{8}$$

There is nothing in (8) to make sure that the sum of the grants so calculated is equal to the share of the pool designated to this part of the scheme. To ensure that such a "balanced budget" condition is satisfied, we use (8) to construct shares that are then applied to the pool. The share for Province i is just its aggregate grant from (8) divided by the sum of the aggregate grants for all Provinces, also from (8). Simplified, this ratio, which defines a share of the pool for Province i, is just:

$$\tau_{i,1} = \frac{P_{i,1}\cdot\gamma_{i,1}}{\sum_i P_{i,1}\cdot\gamma_{i,1}} \qquad 0 \le \tau_{i,1} \le 1 \qquad \textit{(Share of the Pool)} \tag{9}$$

The share of Province i in the growth pool is a function of its population relative to total population (the higher its population the larger is its grant), and its cost disability relative to the sum of all disabilities (the higher is its disability relative to the sum of the disabilities, the greater is its grant). Thus, one can consider this part of the scheme as an equal per capita grant, adjusted for capital cost disabilities. If a Province has a disability of one then it will receive a share of the grant pool based purely on its population share.

Therefore, the aggregate grant for Province i for this component of the scheme is:

$$G^{E}_{i,1} = \tau_{i,1} \cdot CP_1(1-\delta_1) \qquad \textit{(Per capita Grant)} \qquad (10)$$

### *(c) The Aggregate Grant*

The total grant to Province i is the sum of the per capita grant and any domestic backlog grant that may be received:

$$G_{i,1} = G^{E}_{i,1} + G^{D}_{i,1}. \qquad \textit{(Aggregate Grant)} \qquad (11)$$

Provinces with no domestic backlog in period one will receive only a per capita grant. For such Provinces; $G_{i,1} = G^{E}_{i,1}$. Provinces with a positive domestic backlog will receive their per capita grant and a share of the pool allocated to the domestic backlogs (equity). Their share of the backlog pool will depend upon the size of their backlog relative to the backlogs of other Provinces. Once the domestic backlogs have been eliminated all Provinces will receive only the per capita grant (as noted above, once this point is reached, $\delta_t$ will be set equal to zero).

Therefore, policy makers may wish to set $\delta_1$ at a relatively high level in the early stages of the scheme since this implies that most of the pool will go towards eliminating the historical domestic backlogs. The National Government will then meet demands to correct the historical backlogs as quickly as possible and improve equality of access to services. As the scheme progresses, increasingly more emphasis should be placed on the per capita grant by reducing $\delta_1$ as the domestic backlogs diminish.

### 4.2 Period Two

At the commencement of period two we need to adjust the domestic capital backlog estimated in period one to take account of the domestic backlog grant made in period one. Thus, the estimate of the domestic backlog for the commencement of period two is

$$B_{i,2} = B_{i,1} - G^{D}_{i,1}. \qquad \textit{(Historical Domestic Backlog)} \qquad (12)$$

The backlog grant for period two is now based on our estimate of each Province's domestic backlog at the start of period two, namely,

$$G^{D}_{i,2} = \left\{ \begin{array}{ll} \dfrac{B_{i,2}}{\sum\limits_{i:B_{i,2}>0} B_{i,2}} (\delta_2 \cdot CP_2) & \forall \;\; B_{i,2} > 0 \\ 0 & \forall \;\; B_{i,2} \le 0 \end{array} \right\}. \quad \textit{(Historical Backlog Grant)} \qquad (13)$$

As before, the per capita grant is

$$G^{E}_{i,2} = \tau_{i,2} \cdot CP_2 (1 - \delta_2) \qquad \textit{(Per capita grant)} \qquad (14)$$

where

$$\tau_{i,2} = \frac{P_{i,2} \cdot \gamma_{i,1}}{\sum\limits_{i} P_{i,2} \cdot \gamma_{i,1}} \qquad (0 \le \tau_{i,2} \le 1) \qquad \textit{(Share of the pool)}$$

and the total grant to Province i in period two is

$$G_{i,2} = G^{E}_{i,2} + G^{D}_{i,2}. \qquad \textit{(Aggregate Grant)} \qquad (15)$$

In calculating the share $\tau_{i,2}$ we use the cost disability from period one. As will be seen below, this disability is used again for the period three estimates. This seems to be reasonable since it is unlikely that a disability would change significantly between periods. This does not preclude the use of revised estimates in subsequent periods.

### *4.3 Period Three*

For period three the scheme proceeds as before so the following analysis presents the formulas only. The backlog at the beginning of period three is

$$B_{i,3} = B_{i,1} - (G^{D}_{i,1} + G^{D}_{i,2})\,. \qquad (16)$$

The domestic backlog grant for period three is

$$G^{D}_{i,3} = \begin{Bmatrix} \dfrac{B_{i,3}}{\sum\limits_{i:B_{i,3}>0} B_{i,3}} (\delta_3 \cdot CP_3) & \forall \quad B_{i,3} > 0 \\ 0 & \forall \quad B_{i,3} \le 0 \end{Bmatrix}. \qquad (17)$$

The per capita grant is

$$G^{E}_{i,3} = \tau_{i,3} \cdot CP_3 (1 - \delta_3) \qquad (18)$$

where

$$\tau_{i,3} = \frac{P_{i,3} \cdot \gamma_{i,1}}{\sum\limits_{i} P_{i,3} \cdot \gamma_{i,1}} \qquad (0 \le \tau_{i,3} \le 1)$$

and the total grant to Province i in period three is

$$G_{i,3} = G^{E}_{i,3} + G^{D}_{i,3}\,. \qquad (19)$$

### *4.4 Summary*

At the end of the three periods (this could, for example, be nine years if each period is equal to three years corresponding with the MTEF cycle) the historical domestic capital backlog for Province i is just the backlog estimated in period one less the sum of the backlog grants made during the operation of the grant scheme, that is;

$$B_{i,3} = B_{i,1} - \sum_{t=1}^{T} G_{i,t}^{D} \qquad (T = 3). \qquad (20)$$

If at the completion of three periods the Provinces still have domestic backlogs, as measured by (20), then the backlog part of the grant scheme would require a second phase. This would be the same as the first three-period phase, but would use, as its initial backlog estimates, the number(s) derived from (20). All Provinces would continue to receive the per capita grant on the assumption that there is still a need to raise the general level of public capital formation in the Provinces.

Also note that we have allowed, in the analysis above, the cost disabilities to affect both components of the grant scheme, namely, the per capita part and the domestic backlogs. However, in the simulation model constructed from the formulas above, we have allowed for the possibility that the policy maker may wish to allow disabilities to affect only one part of the grant allocation (eg. the per capita part) or have no influence on the allocations at all (essentially, by setting all the betas equal to zero in the program). In this way, we have allowed for maximum flexibility.

## 5. Preliminary Simulations With and Without Disabilities

Having provided a detailed conceptual discussion of the model and cost disabilities, we now discuss the simulation model and present the results of some simulations. The analysis is presented as follows. First, is a brief discussion of the input database. This is followed by a discussion of how the disabilities indexes are constructed and the interaction between disabilities. Finally, there is a discussion of simulations and the implications of including/excluding disabilities.

Before commencing the analysis there are several things to be aware of. First, the simulation model can be run without the cost disabilities affecting the results. This is achieved by setting all of the betas equal to zero. Thus, the policy maker has the option of allocating grants to the Provinces with or without disabilities being accounted for. Second, the disability inclusive simulations presented below should be seen as preliminary since they are based on the inclusion of four disabilities which may be revised between now and the final version of the model (some may be deleted/revised and others may be included in the disability matrix), and because they are generated using preliminary basic input data which may also be revised/refined. Thus, the simulations in this Section should be seen as illustrative only.

### *5.1 Input Data Base*

The simulation model requires data on a variety of economic indicators in South Africa. Before looking at the model in detail it is important to summarise the basic data inputs and their sources (all the basic in put data was supplied to us by the FFC).

The first key requirement is an estimate of the total provincial government capital stock, both at the aggregate economy level and disaggregated to the Provincial level. The basic data, which was provided to the FFC by the South African Reserve Bank (SARB), is in constant 1995 prices for the years 1990 to 2002. The Reserve Bank provides this data both in aggregate form and disaggregated at the Provincial level using underlying capital flows as weights to disaggregate the data. As well as using the SARB weights to disaggregate the capital stock the model database is also able to calculate weights based on Treasury figures on capital expenditure by province from 1995/96 to 2000/01. The simulation model allows the user to choose between the weights or use an average of the two[11]. For the current simulations we used an average of the two sets of weights, these are presented in table 2 and are the weights which were used to disaggregate the total 2002 capital stock of some R 62,406 million amongst the Provinces in Table 1.

[11] One re-assuring aspect of the data is that the two sets of weights, those from SARB and those based on Treasury data are very similar.

**Table 2: Weights used to Allocate**
**Total Capital Stock to Each Province**

| | Average of SARB and Treasury |
|---|---|
| WC | 11.1% |
| EC | 6.5% |
| NC | 2.8% |
| FS | 4.9% |
| KZN | 28.2% |
| NW | 9.6% |
| GA | 20.6% |
| MP | 8.7% |
| LP | 7.6% |

We should note in passing the importance of these figures. Take as an example KZN, on the basis of the data above the model will allocate 28.2 per cent of the total capital stock to this province. When the backlog is calculated as the difference between the desired capital stock (based itself on the S.A. average) and the measured capital stock then clearly this figure will play a significant role in determining whether a Province has a backlog.

The second key data requirement is population data. Figures on total population, both in aggregate and at the Province level, were provided to us by the FFC. They are, in turn, sourced from Stats S.A: in particular, the current estimate of mid year populations for 2002 comes from: *Statistical release P0302 "Mid Year Estimates 2002" Statistics South Africa.* Population figures are important in the model for two reasons. Firstly, in the base year the backlogs are calculated using an estimate of the amount of Capital per person[12] and secondly because, over the simulation horizon, the program requires an estimate of the Provincial populations in order to allocate the per capita portion of the pool of funds. In order to forecast over the simulation horizon the input database uses the implied exponential growth rates for males and females at the required level of disaggregation for the period 1996 to 2002 in Table E of Statistical release P0302. These inferred growth rates include provision for inter-provincial migration and the apparent under reporting of

[12] Recall from Table 1 that based on an estimated capital stock of R 62,407 million and a population of 45.17 million this gave an estimate of R1382 per person

mortality rates in non urban regions. Stats SA has also attempted to calculate the growth rates when allowance is made in the figures for additional deaths due to HIV/AIDS. These figures are based on estimates of infection rates from antenatal clinic surveys. In the simulations below we chose to use the growth rates based on the assumption that the effects of AIDs were included in the calculations.

The final data input is the projected size of the total pool of funds available over the three periods of the simulation. The input database allows this pool to be calculated in a variety of ways, including basing it on a forecast of GDP over the simulation period. However, for simplicity we chose to over-ride this option and input the amount manually. Based on discussions with the FFC, we selected for the current simulations a pool size of R 18,000 million (2003 prices) over the three periods of the simulation, some R 6,000 per period. Since all calculations are carried out in real terms we also had to assume some inflation rate over the three periods, we chose 8% in period 1, 7% in period 2 and 6% in period 3.

### *5.2 Constructing Disabilities*

The method for constructing the disability variable $\gamma_i$ is explained by equations (1) and (2). Two pieces of information are required for the construction of the mean deviations. The first is a set of betas; the weights attached to each disability. In the model, the betas are determined by the policy maker. We will describe their effect and our choice, both below, and in the next Section when we discuss the simulations. The second set of information is data on the actual disabilities and we describe these below. This data was supplied by the FFC and at present is in draft form.

Currently, there are four disabilities in the model and all are used in the current preliminary simulations. However, as noted earlier, the user can switch off any particular disability by setting the associated beta value equal to zero. If one sets all betas equal to zero then the effect of all of the disabilities is taken out of the simulations. Also, one can easily incorporate additional disabilities within the separate program that constructs the disability matrix (without altering the main program which contains the formulas). Indeed, between now and the final version it is likely that the disabilities will be revised in various ways (eg. inclusion of new disabilities and/or discarding some of those constructed below for the purposes of these preliminary simulations).

### *(a) Disability measure 1: Income Inequality*

The argument here is that provinces with large proportions of households living in poverty have a positive disability and that the capital grant model should allocate proportionately more money to such provinces. We use data on household income at the provincial level to construct a measure which reflects the proportion of Provinces households which have incomes below some chosen threshold. The data, which is sourced from Statistics SA shows the number of households in a range of income groups for each province and is further subdivided according to whether the household unit was "urban" or "non-urban". To construct the measure we simply calculate the proportion of the total number of household units in the province with incomes below some selected level which the model allows the user to select. As an example and for the simulations below we chose to use households with incomes less than or equal to a maximum of R4800.

Table 3 shows the key results for disability 1. In column 2 we have the disability measures, the X's in the formula, thus for Western Cape our data tells us that 14.7 per cent of the population in the Province have an income level below R4800, in Limpopo our data suggests that the figure is 43.3 per cent. Clearly then logic tells us that the disability weight for this particular disability should be higher for Limpopo than Western Cape. From the X's we can calculate the mean value of the disability measure for all Provinces (31.798 in this case). This then allows us to calculate the mean deviations for each province (presented in column 3).

**Table 3: Key Data For Disability One**

| | Disability Measure (X) | %Deviation of X from mean value (D) | Disability Weights ($\gamma_i$) | Provinces Pop. As proportion of Total Pop | $\tau_i$ for Period 1 |
|---|---|---|---|---|---|
| WC | 14.71 | -53.8% | 0.76 | 9.5% | 7.1% |
| EC | 40.13 | 26.2% | 1.14 | 15.8% | 17.6% |
| NC | 22.98 | -27.7% | 0.87 | 1.9% | 1.7% |
| FS | 37.32 | 17.4% | 1.09 | 6.3% | 6.7% |
| KZN | 35.51 | 11.7% | 1.06 | 20.3% | 21.0% |
| NW | 34.19 | 7.5% | 1.04 | 8.1% | 8.2% |
| GT | 23.67 | -25.6% | 0.88 | 17.9% | 15.4% |
| MP | 34.39 | 8.2% | 1.04 | 7.0% | 7.1% |
| LP | 43.31 | 36.2% | 1.20 | 13.1% | 15.3% |

So, for example, the Table tells us that Western Cape has a proportion of households falling into the selected income category which is some 53.8 percent below the average (approximately (14.7 –31.8)/31.8 = -0.538) whilst Limpopo has 36.2 percent more than the average. For this particular disability then 6 out of the 9 provinces are at a "disadvantage" compared to the average (EC, FS, KZN, NW, MP and LP, ie all those with a positive entry in the table above), these are the provinces whose disability measure is above the average figure[13]. Having calculated the D values for this particular disability we can choose some value of $\beta$ and use this to calculate the cost disability variable $\gamma$, *assuming that this were the only disability*. We chose to set $\beta = 0.5$ for all the calculations presented in this section, this gives us the disability weights in column 4. As discussed earlier, the exponential transformation means that the cost disability variable is normalized around 1, with provinces with positive disabilities (such as Limpopo) being given values in excess of 1 (1.19 and the highest for Limpopo) and provinces with negative disabilities (such as Western Cape) being given values below 1 (0.76 and the lowest for Western Cape).

[13] In programming the disabilities into the FFC's Provincial Capital Grant Model we have chosen to measure all disabilities in this way such that larger values of the X's will reflect positive disabilities.

It is worth recalling the role played by $\gamma_i$ in the model calculations. Firstly the earlier discussion shows us that $\gamma_i$ plays a role in the calculation of the desired capital stock and hence the domestic backlog. Thus, a province with a positive disability will have a value of $\gamma_i$ in excess of 1. This will raise the desired capital stock and increase the size of any backlog.

The earlier discussion also showed us that $\gamma_i$ will also influence the per capita allocation. It is worth, therefore, calculating $\tau$ using the example that we have. To do this we need both the provincial population and the total population and in Table 3 we have used population figures for 2002 - the fifth column of Table 3 shows each province's population as a proportion of the total, thus for example Western Cape has approximately 9.5 per cent of the population whilst Limpopo has 13.1 percent. If a province happened to have a zero disability (ie; its X value is equal to the average) then its measured $\gamma$ would equal one[14] and the calculated $\tau$ would simply equal the population in the province as a proportion of the total population. So a comparison of columns 5 and 6 of the table allows us to see the effect of the disabilities on the per capita allocation. Because Limpopo had a positive disability its share in this part of the pool is raised from 13.1 percent to 15.3 percent whilst Western Capes share falls from 9.5 per cent to 7.1 per cent. We now briefly describe each of the other three disability measures currently used in the model and reproduce Table 3 for each so that we can see which provinces have disability weights above 1 in value and which have weights below 1 in value. In all calculations a beta ($\beta$) of 0.5 was used.

***(b) Disability measure 2: Ratio of Rural to Urban Income/Population Density.***

The second disability focuses on the idea that the cost of capital provision is higher in provinces with higher proportions of non-urban populations or Provinces where the population is low density. Several of the data sources in the FFC data warehouse could provide a basis for calculation of this measure. For example one possibility would be to

[14] Since $e^0 = 1$.

calculate km$^2$ per person[15] in each of the Provinces and use this as a disability measure. One obvious problem with such a measure however is the fact that large proportions of some provinces may actually be uninhabited, thus biasing the measure upwards. A second possibility, and the one we have used here, is simply to use the previously noted data which, for each province, allows calculation of the total number of households in urban and non urban regions. Table 4 reports the proportion of households which are classified as non-urban by province and uses this as the disability measure X.

**Table 4: Key Data For Disability Two**

| | Disability Measure (X) | %Deviation of X from mean value (D) | Disability Variable ($\gamma_i$) | Provinces Pop. As proportion of Total Pop | $\tau_i$ for period 1 |
|---|---|---|---|---|---|
| WC | 9.71% | -75.8% | 0.68 | 9.5% | 6.0% |
| EC | 57.02% | 42.0% | 1.23 | 15.8% | 18.1% |
| NC | 23.67% | -41.1% | 0.81 | 1.9% | 1.5% |
| FS | 21.22% | -47.2% | 0.79 | 6.3% | 4.6% |
| KZN | 46.97% | 16.9% | 1.09 | 20.3% | 20.5% |
| NW | 57.78% | 43.9% | 1.25 | 8.1% | 9.3% |
| GT | 3.49% | -91.3% | 0.63 | 17.9% | 10.5% |
| MP | 54.68% | 36.1% | 1.20 | 7.0% | 7.8% |
| LP | 86.91% | 116.4% | 1.79 | 13.1% | 21.7% |

Thus, for example, in Western Cape, out of a total of 1,173,303 reported households only 113,907, some 9.7 per cent, were classified as non-urban, in Eastern Cape however the proportion was 57.0 percent. Column 3 reports the calculated D and as can be seen there is quite a large range for this disability. Once again Limpopo has the largest disability, some 116.4 percent above the average (86.9 per cent of the households were classified as non-urban) and Gauteng has the largest negative disability,  some 91.3 per cent below the average (only 3.5 per cent of the households were classified as non-urban).

[15] Note that this is the inverse of the more familiar persons per sq. km. and would be our preferred measure since we want higher values of the measure to reflect greater disabilities.

For this disability five provinces (EC, KZN, NW, MP and LP) have $\gamma_i$ measures in excess of 1 whilst 4 (WC, NC, FS and GT) have values below 1. Finally, columns 5 and 6 show the share of the population by province, and the calculated values of $\tau_i$ for comparison purposes.

### *(c) Disability measure 3: Debilitating Disease (Example: HIV Infection Rates)*

Disability measure 3 is based on the impact of a debilitating disease indicator. In South Africa TB, malnutrition and HIV/Aids all have significant impacts across provinces. Data for these three diseases is currently being assessed for reliability and use in our simulations. For illustrative purposes only we use the available data for HIV/Aids fully aware of the problems associated with the methods for estimating this data. In this example the model will assess the impact that the spread of the AID's virus will have on the cost of health care provision in the provinces. Once again, the key concept is simple, higher rates of HIV infection will lead to provinces incurring higher costs of provision of health services to deal with the virus. We used data from the Department of Health Summary Report. *National HIV and Syphilis Sero-Prevalence Survey in South Africa, 2001.* The survey shows HIV prevalence rates for the years 1994 to 2001. These figures are reported in column 2 of Table 5, by province.

**Table 5: Key Data for Disability Measure Three**

| | Disability Measure (X) | %Deviation of X from mean value (D) | Disability Variable ($\gamma_i$) | Provinces Pop. As proportion of Total Pop | $\tau_i$ period 1 |
|---|---|---|---|---|---|
| WC | 8.6 | -62.9% | 0.73 | 9.5% | 6.0% |
| EC | 21.7 | -6.3% | 0.97 | 15.8% | 18.1% |
| NC | 15.9 | -31.4% | 0.85 | 1.9% | 1.4% |
| FS | 30.1 | 29.9% | 1.16 | 6.3% | 4.6% |
| KZN | 33.5 | 44.6% | 1.25 | 20.3% | 20.5% |
| NW | 25.2 | 8.8% | 1.04 | 8.1% | 9.3% |
| GT | 29.8 | 28.6% | 1.15 | 17.9% | 10.5% |
| MP | 29.2 | 26.0% | 1.14 | 7.0% | 7.8% |
| LP | 14.5 | -37.4% | 0.83 | 13.1% | 21.7% |

The highest infection rate appears to be KZN with some 33.5 percent whilst the lowest rate is in Limpopo with 14.5 per cent. Five provinces have $\gamma_i$ values exceeding 1, due to the fact that their infection rates are above the South African average. These provinces are FS, KZN, NW, GT and MP whilst four fall below 1, WC, EC NC and LP. One can see from this that KZN has the largest disability for the Aids disability measure.

***(d) Disability measure 4: Unemployment.***

The final set of data uses measured unemployment rates in the provinces arguing that high unemployment brings with it socio-economic disabilities which should be taken account of in the calculation of capital allocations. We used "Statistical release P0210" – from StatsSA which provides provincial unemployment rates and we have used these to measure disabilities, the basic data are reported in Table 6.

**Table 6: Key Data for Disability Measure Four**

| | Disability Measure (X) | %Deviation of X from mean value (D) | Disability Weights ($\gamma_i$) | Provinces Pop. As proportion of Total Pop | $\tau_i$ period 1 |
|---|---|---|---|---|---|
| WC | 20.6 | -26.7% | 0.87 | 9.5% | 8.2% |
| EC | 31.8 | 13.1% | 1.07 | 15.8% | 16.7% |
| NC | 27.5 | -2.2% | 0.99 | 1.9% | 1.9% |
| FS | 28.6 | 1.7% | 1.01 | 6.3% | 6.3% |
| KZN | 31.3 | 11.3% | 1.06 | 20.3% | 21.2% |
| NW | 29.4 | 4.6% | 1.02 | 8.1% | 8.1% |
| GT | 28.2 | 0.3% | 1.00 | 17.9% | 17.7% |
| MP | 25 | -11.1% | 0.95 | 7.0% | 6.5% |
| LP | 30.6 | 8.9% | 1.05 | 13.1% | 13.5% |

In this example, EC has the highest unemployment rate at 31.8 per cent, whilst WC has the lowest at 20.6. Table 6 calculates the values of D and the $\gamma$'s again on the basis that we have a single disability and that $\beta = 0.5$. Since the average unemployment rate is 28.11 per cent, based on these figures 6 Provinces have positive disabilities, EC, FS, KZN, NW, GT and LP whilst the remaining 3, WC, NC and MP have negative disabilities. Finally, Table 6 reports the calculated $\tau$'s. In this particular case, since the variance of unemployment rates by province is relatively small (none face a significantly different disability) the disability measure is relatively close to 1 for all provinces and would therefore have only a minor effect on the grant allocation.

### *(e) The Disability Measures Together*

In practice, the disability measures will be used as a group with potentially different weights selected for each disability using the $\beta$'s[16]. Therefore, we now present the calculated disability measures, using the data described above, and assuming that $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0.5$. Table 7 presents the key results including the value of $\gamma_i$ when constructed using all the disabilities.

**Table 7: The Overall Disability for Each Province**

| | Key Variables | | |
|---|---|---|---|
| | $\gamma_i$ | Provinces pop. as proportion of total population | $\tau_i$ (period 1) |
| WC | 0.33 | 9.5% | 2.6% |
| EC | 1.45 | 15.8% | 19.0% |
| NC | 0.60 | 1.9% | 0.9% |
| FS | 1.01 | 6.3% | 5.25% |
| KZN | 1.53 | 20.3% | 25.6% |
| NW | 1.38 | 8.1% | 9.2% |
| GT | 0.64 | 17.9% | 9.5% |
| MP | 1.35 | 7.0% | 7.8% |
| LP | 1.86 | 13.1% | 20.1% |

Six provinces would have weightings of greater than 1 implying positive disabilities. These are, in order of magnitude, Limpopo (1.86), KwaZulu-Natal (1.53), Eastern Cape (1.45), North West (1.38), Mpumalanga (1.34) and Free State (1.01). For these provinces, the value of $\tau_i$ (the share in the per capita growth related grant allocation) will be greater than their population share. For example, because EC has a disability greater than one its share of the per capita part of the grant will be greater than its population share. On the other hand, three provinces receive weightings less than 1: Gauteng (0.64),

[16] Using a $\beta$ of zero, therefore, allows the policy maker to exclude a disability from the simulation.

Northern Cape (0.60) and Western Cape (0.33). The implication is that their share of the per capita component of the grant pool is less than their population shares.

### *5.3 Preliminary Simulations*

In this section we provide an example of the effects of inclusion of disabilities in the calculation of the capital grants. We run the model using the set of assumptions discussed above and since it is vital to understand these assumptions in order to make sense of the results of the model we list them briefly below in point form:

- The capital stock weightings (the weights used to allocate the capital stock between the provinces) are based on an average of Treasury and SARB weights (see Table 2).
- Population forecasts (based on StatsSA data) can either include or exclude the effects of AIDS on the population. We chose to include the effects.
- The model allows the user to either allow the capital backlog calculations to include the effects of population change over the simulation period, or exclude it. We chose to exclude the effects of population change.
- The size of the total grant pool available can be determined in a number of ways. We chose to determine the pool exogenously at R 6000 Millions in 2003 current prices for each of the three simulation periods[17] giving a total of R18,000 Million.
- The parameter $\delta$ needs to be set over the three periods. Recall that this determines the proportion of the total pool allocated to the backlogs component, ($1-\delta$) going to the efficiency component. This was set at 0.8 in period one, 0.7 in period two and 0.6 in period three.

This is the set of assumptions required to run the model excluding disabilities, however once disabilities are included we need the following, all of which have been discussed above.

[17] Note that over the simulation period inflation will reduce the real value of the grant in future periods.

- The betas ($\beta$) which weight the disabilities need to be set. We chose to set $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0.5$.
- For disability 1, income inequality, we chose to use the proportion of households with income below R 4800.
- For disability 2, the ratio of rural to urban income / population density we chose to use the proportion of non urban households in the province.
- For disability 3, AIDS infections, we chose to use to use the HIV prevalence rates discussed above.
- For Disability 4, unemployment rate, we chose to use recently released data on unemployment rates.

Two simulations were then carried out; one excluding the disabilities from the calculations, and one including them. Table 8 shows the effect of including the disabilities on the domestic backlogs. The key element in the Table is the entry "Effect on Backlog". If this is positive, then the province's calculated backlog has risen (or its surplus is smaller): if it is negative its backlog is smaller (its surplus is bigger).

Let us take Eastern Cape as an example. If the disabilities are excluded from the calculation Eastern Cape has a backlog of R 5,795 Million, however, because its disability weight is 1.45 this has the effect in the model of making the optimal capital stock larger – increasing it from R 9,853[18] million to R 14,334 million and hence making its capital backlog even larger at R 10,276 million. The logic is clear; because Eastern Cape faces substantial disabilities in the provision of capital it requires a greater stock of capital. On the other hand, Northern Cape, without disabilities, has a negative backlog (surplus) of R 516 Million. Since its disability weight is below 1 (ie 0.6) the effect of introducing disabilities is to raise its surplus to R 736 Million[19].

[18] Note that all figures in this section are in 1995 constant prices as the model carries out all calculations of backlogs in real 1995 prices as this was the base year used in the data supplied by SARB.

[19] We should note that Provinces with surpluses simply have their shares in the backlog component of the grant set to zero, they are not penalized for the relative size of any calculated surplus.

Overall, if we exclude disabilities the total domestic backlog (ignoring surplus Provinces) is R 9,987 Million (approx R 17,916 Million in 2003 prices). However, if we include disabilities, the aggregate domestic backlog rises to R 24,691 million (approx. R 44,292 Million in 2003 prices). The most obvious outcome, however, is that the number of provinces sharing in the backlog component of the grant rises from three to six, Eastern Cape, Free State and Limpopo being joined by Kwa-Zulu Natal, North West and Mpumalamga.

**Table 8: Desired Capital Stocks and Domestic Backlogs in Period One, Excluding and including Disabilities, 1995 Constant Prices.**

| | |
|---|---|
| Total Provincial Capital Stock (real 1995 prices, Million) | 62,406 |
| Total pop. (Million) | 45.17 |

| | | | | | |
|---|---|---|---|---|---|
| WC Capital Stock (real 1995 prices) | 6,912 | EC Capital Stock (real 1995 prices) | 4,058 | KZN Capital Stock (real 1995 prices) | 17,608 |
| WC population | 4.31 | EC population | 7.13 | KZN population | 9.21 |
| Optimal Cap Stock (no disabilities) | 5,960 | Optimal Cap Stock(no disabilities) | 9,853 | Optimal Cap Stock(no disabilities) | 12,727 |
| WC Backlog (no disabilities) | -952 | Backlog(no disabilities) | 5,795 | Backlog(no disabilities) | -4,881 |
| Optimal Cap Stock with disabilities | 1,992 | Optimal Cap Stock with disabilities | 14,334 | Optimal Cap Stock with disabilities | 19,425 |
| Backlog with disabilities | -4,920 | Backlog with disabilities | 10,276 | Backlog with disabilities | 1,817 |
| Effect on Backlog | **-3,968** | | **+4,481** | | **+6,699** |
| | | | | | |
| NC Capital Stock (real 1995 prices) | 1,743 | FS Capital Stock (real 1995 prices) | 3,086 | NW Capital Stock (real 1995 prices) | 6,018 |
| NC population | 0.89 | FS population | 2.86 | NW population | 3.66 |
| Optimal Cap Stock(no disabilities) | 1,227 | Optimal Cap Stock(no disabilities) | 3,950 | Optimal Cap Stock(no disabilities) | 5,056 |
| Backlog(no disabilities) | -516 | Backlog(no disabilities) | 864 | Backlog(no disabilities) | -962 |
| Optimal Cap Stock with disabilities | 736 | Optimal Cap Stock with disabilities | 3,987 | Optimal Cap Stock with disabilities | 6,989 |
| Backlog with disabilities | -1,007 | Backlog with disabilities | 901 | Backlog with disabilities | 971 |
| Effect on Backlog | **-492** | Effect on Backlog | **+37** | Effect on Backlog | **+1,933** |
| | | | | | |
| G Capital Stock (real 1995 prices) | 12,832 | MP Capital Stock (real 1995 prices) | 5,405 | LP Capital Stock (real 1995 prices) | 4,745 |
| G population | 8.11 | MP population | 3.16 | LP population | 5.84 |
| Optimal Cap Stock(no disabilities) | 11,199 | Optimal Cap Stock(no disabilities) | 4,360 | Optimal Cap Stock(no disabilities) | 8,073 |
| Backlog(no disabilities) | -1,633 | Backlog(no disabilities) | -1,044 | Backlog(no disabilities) | 3,328 |
| Optimal Cap Stock with disabilities | 7,214 | Optimal Cap Stock with disabilities | 5,865 | Optimal Cap Stock with disabilities | 15,011 |
| Backlog with disabilities | -5,617 | Backlog with disabilities | 460 | Backlog with disabilities | 10,265 |
| | **-3,985** | | **+1,504** | | **+6,937** |

Figure 2 summarises the results from Table 8 graphically. Eastern Cape, Limpopo and Free State all have their domestic backlogs increased through the inclusion of disabilities.

However, they are, as noted, now joined by KwaZulu-Natal with a significant disability, Mpumulanga and North West. Clearly, since more provinces are now sharing in the domestic backlog pool, and because the total backlogs are larger, this will have implications for the shares, and for the speed with which the backlog is eroded.

**Figure 2: Calculated Domestic Backlogs, Period One**

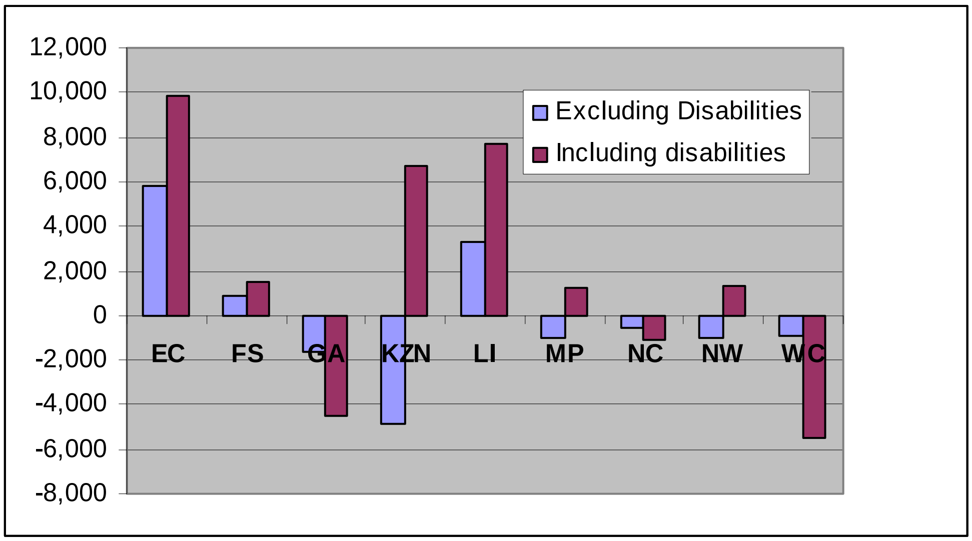


Table 8a further illustrates the impact of disabilities on the provinces' shares of the backlog grant pool for period one. Two Provinces have their share in the backlog pool reduced. One is Eastern Cape which sees its share fall from 58 per cent to 41.6 per cent and the other is Free State: its share falls from 8.7 per cent to 3.7 per cent. Limpopo, because of its high disability weight, gains (from 33.3 per cent to 41.6 per cent) and of course the additional three provinces now take a share with KwaZulu-Natal being the biggest beneficiary, receiving 7.3 per cent of the domestic backlog pool.

**Table 8a: Shares of Each Province in Domestic Backlog Pool, Period One**

| | WC | EC | NC | FS | KZN | NW | GT | MP | LP |
|---|---|---|---|---|---|---|---|---|---|
| Excluding Disabilities | | 58.0% | | 8.7% | | | | | 33.3% |
| Including Disabilities | | 41.6% | | 3.7% | 7.3% | 3.9% | | 1.9% | 41.6% |

Tables 8b and 8c show how these shares in the domestic backlog pool convert into Rands (in 2003 constant prices), first excluding disabilities (Table 8b), and second, including disabilities (Table 8c)[20].

**Table 8b: Backlog Grants (R Million), Excluding Disabilities**

| Backlog Grant R million constant 2003 prices | | | | | | | | | | Total backlog Grant |
|---|---|---|---|---|---|---|---|---|---|---|
| | WC | EC | NC | FS | KZN | NW | GT | MP | LP | |
| period 1 | 0 | 2785.2 | 0 | 415.3 | 0 | 0 | 0 | 0 | 1599.6 | 4800.0 |
| period 2 | 0 | 2277.6 | 0 | 339.6 | 0 | 0 | 0 | 0 | 1308.1 | 3925.2 |
| period 3 | 0 | 1841.7 | 0 | 274.6 | 0 | 0 | 0 | 0 | 1057.8 | 3174.0 |

**Table 8c: Domestic Backlog Grants (Rand Million), Including Disabilities**

| Backlog Grant R million constant 2003 prices | | | | | | | | | | Total backlog Grant |
|---|---|---|---|---|---|---|---|---|---|---|
| | WC | EC | NC | FS | KZN | NW | GT | MP | LP | |
| period 1 | 0 | 1997.7 | 0 | 175.2 | 353.3 | 188.8 | 0 | 89.5 | 1995.6 | 4800.0 |
| period 2 | 0 | 1633.6 | 0 | 143.2 | 288.9 | 154.4 | 0 | 73.1 | 1631.9 | 3925.2 |
| period 3 | 0 | 1321.0 | 0 | 115.8 | 233.6 | 124.9 | 0 | 59.1 | 1319.6 | 3174.0 |

Table 9 illustrates the impact of the disabilities on the speed with which domestic backlogs are reduced. The Table shows the domestic backlog at the beginning of each period; so in the row entitled period 2, the figure shows the backlog at the beginning of period 2 after period 1's backlog grant to the Provinces. Since delta ($\delta$) is set at 0.8 and the period 1 total pool is R 6000 Million, this shows the backlog declining by R4,800

[20] The Total Pool size was set at R 18,000 Million in 2003 current prices and spread evenly over the three periods. In real terms the pool in subsequent years is eroded in by the assumed rate of inflation. In the calculations this is important as the backlog is calculated and set in real terms at the beginning of the simulation. Also note that the parameter delta ($\delta$) falls from 0.8 in period 1 to 0.7 in period 2 to 0.6 in period 3 so that proportionately more of the pool is allocated to the per capita portion.

Million. Note that in subsequent periods the reduction is affected by the erosion of the grant value due to the assumed inflation rate. For a given pool of funds, the inclusion of disabilities, because it raises the total domestic backlog (by raising the optimal capital stock in provinces with backlogs and bringing other provinces into the backlogs part of the scheme), has the effect of significantly slowing down the process of backlog elimination.

**Table 9: Backlog Reduction over the Simulation Periods**

| | Total Backlog beginning of period in 2003 constant prices, Rand Million | Backlog reduction per period | Backlog Reduction cumulative |
|---|---|---|---|
| **Excluding Disabilities** | | | |
| Period 1 | 17,916 | | |
| Period 2 | 13,116 | 27% | 27% |
| Period 3 | 9,191 | 30% | 49% |
| Period 4 | 6,017 | 35% | 66% |
| **Including Disabilities** | | | |
| Period 1 | 44,293 | | |
| Period 2 | 39,493 | 11% | 11% |
| Period 3 | 35,568 | 10% | 20% |
| Period 4 | 32,394 | 9% | 27% |

Finally, we have used $\tau$ to illustrate in earlier discussion how the disabilities affect the per capita part of the grant scheme. Table 10a illustrates this over the three periods, showing each Province's percentage share in the per capita grant pool. The Rand Million amounts, based on the assumptions made above, are provided in Table 10b for the case where disabilities are included. Table 10c shows the population forecasts by Province used in the model and how the $\tau$'s change over the simulation period (for the case where we include disabilities). Over the three periods there is little variation in the calculated $\tau$'s: the changes that do occur are the result of changes in the population shares.

**Table 10a: Provincial Percentage Shares in the Per Capita Grant Pool**

| **Excluding Disabilities** | WC | EC | NC | FS | KZN | NW | GT | MP | LP |
|---|---|---|---|---|---|---|---|---|---|
| period 1 | 9.5% | 15.8% | 1.9% | 6.3% | 20.3% | 8.1% | 17.9% | 7.0% | 13.1% |
| period 2 | 9.5% | 15.9% | 1.9% | 6.3% | 20.3% | 8.0% | 17.9% | 7.0% | 13.2% |
| period 3 | 9.4% | 15.9% | 1.9% | 6.2% | 20.2% | 8.0% | 17.9% | 7.0% | 13.4% |
| **Including Disabilities** | WC | EC | NC | FS | KZN | NW | GT | MP | LP |
| period 1 | 2.6% | 19.0% | 1.0% | 5.2% | 25.6% | 9.2% | 9.5% | 7.8% | 20.1% |
| period 2 | 2.6% | 19.0% | 1.0% | 5.2% | 25.5% | 9.2% | 9.5% | 7.8% | 20.3% |
| period 3 | 2.6% | 19.1 | 0.9% | 5.2% | 25.4% | 9.1% | 9.5% | 7.8% | 20.5% |

**Table 10b: Provincial Shares in Per Capita Grant Pool Rand Million (2003 constant Prices)**

| **Excluding Disabilities** | WC | EC | NC | FS | KZN | NW | GT | MP | LP | Total R'm 2003 current prices |
|---|---|---|---|---|---|---|---|---|---|---|
| period 1 | 114.2 | 190.0 | 23.4 | 75.6 | 244.0 | 96.9 | 215.0 | 84.0 | 157.0 | 1200 |
| period 2 | 159.5 | 267.1 | 32.5 | 105.5 | 341.0 | 135.3 | 300.9 | 118.0 | 222.4 | 1682 |
| period 3 | 199.8 | 336.9 | 40.5 | 132.1 | 427.6 | 169.6 | 377.8 | 148.8 | 282.9 | 2116 |
| **Including Disabilities** | WC | EC | NC | FS | KZN | NW | GT | MP | LP | Total R'm 2003 current prices |
| period 1 | 31.5 | 228.0 | 11.6 | 63.0 | 307.2 | 110.5 | 114.3 | 93.2 | 240.8 | 1200 |
| period 2 | 43.9 | 320.2 | 16.0 | 87.8 | 428.9 | 154.1 | 159.7 | 130.8 | 340.8 | 1682 |
| period 3 | 55.0 | 403.4 | 20.0 | 109.8 | 537.1 | 192.9 | 200.3 | 164.7 | 432.9 | 2116 |

**Table 10c:Population Forecasts and Calculated τ's (Including Disabilities)**

| | Popn Period 1 | Popn Period 2 | Popn Period 3 | $\tau_i$ period 1 | $\tau_i$ period 2 | $\tau_i$ period 3 |
|---|---|---|---|---|---|---|
| WC | 4.38 | 4.45 | 4.51 | 0.02623717 | 0.02610804 | 0.0259787 |
| EC | 7.29 | 7.45 | 7.61 | 0.19002685 | 0.19032773 | 0.1906232 |
| NC | 0.90 | 0.91 | 0.91 | 0.00963865 | 0.00953983 | 0.00944171 |
| FS | 2.90 | 2.94 | 2.98 | 0.05247327 | 0.05217648 | 0.05187966 |
| KZN | 9.36 | 9.51 | 9.66 | 0.25602604 | 0.25493744 | 0.25384506 |
| NW | 3.72 | 3.77 | 3.83 | 0.09206329 | 0.09162015 | 0.09117612 |
| GT | 8.25 | 8.39 | 8.53 | 0.09521611 | 0.09494261 | 0.09466694 |
| MP | 3.22 | 3.29 | 3.36 | 0.07769335 | 0.07775932 | 0.07782276 |
| LP | 6.02 | 6.20 | 6.39 | 0.20062527 | 0.2025884 | 0.20456584 |
| Total Popn | 46.03 | 46.90 | 47.80 | | | |

### *5.4 Summary*

The analysis above makes a case for the need to incorporate disabilities into the FFC's capital grant allocation model. The current simulations suggest that the inclusion of disability weights will, as would be expected, alter the shape of the grant distribution amongst the Provinces of South Africa. It is clear that the inclusion of disabilities has a substantial effect on the model, with the current disability measures and a conservative setting of the weighting matrix, six of the provinces now show evidence of having backlogs and share in the backlog pool. It is also clear that the disability measures and the weighting matrix are important parts of the policy choice and that thought needs to go into the setting of these weights.

## 6. Conclusion

The paper has described how Province specific capital cost disabilities can be derived from the notion of cost functions used in economics. It has also been shown that differences in population dispersion, or the incidence of diseases such as Aids, can differentially affect the minimum cost of providing a given level of public service output between Provinces. Furthermore, we have argued that cost disabilities can be constructed, amalgamated into a disability function and used to construct an aggregate disability variable for each Province.

The paper has also shown how one can construct a grant model that allocates a capital grant pool to the Provinces of South Africa in two components. The first is a per capita allocation to all Provinces. Here the particular grant to each Province is dependent on its population share and its disability. The second type of allocation is one that is designed to address historical capital backlogs. The share of a Province in this part of the grant pool is determined by a comparison of the capital stock that we would like the Province to have, based on a domestic standard, and the capital stock that it actually has, adjusted for cost disabilities.

Finally, the paper has discussed how the formulas of the model can be constructed within a computer simulation model, and has presented the results of two preliminary simulations based on various assumed parameter values.

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